Ensemble Kalman Filter with a Square Root Scheme (EnKF-SR) for Trajectory Estimation of AUV SEGOROGENI ITS

Teguh Herlambang[1], Eko Budi Djatmiko[2], Hendro Nurhadi[3]

Abstract – Results of a study on the development of navigation system and guidance for AUV are presented in this paper. The study was carried to evaluate the behavior of AUV *Segorogeni ITS*, designed with a characteristic length of 980 mm, cross-section diameter of 180 mm, for operation in a 3.0 m water depth, at a maximum forward speed of 1.94 knots. The most common problem in the development of AUVs is the limitation in the mathematical model and the restriction on the degree of freedom in simulation. In this study a model of linear system was implemented, derived from a non-linear system that is linearized utilizing the Jacobian matrix. The linear system is then implemented as a platform to estimate the trajectory. In this respect the estimation is carried out by adopting the method of Ensemble Kalman Filter Square Root (EnKF-SR). The EnKF-SR method basically is developed from EnKF at the stage of correction algorithm. The implementation of EnKF-SR on the linear model comprises of three simulations, each of which generates 100, 200 and 300 ensembles. The best simulation exhibited the error between the real tracking and the simulation in translation mode was in the order of 0.009 m/s, whereas in the rotation mode was some 0.001 rad/s. These fact indicates the accuracy of higher than 95% has been achieved.

***Keywords**:* AUV, EnKF-SR, Linear system, Trajectory Estimation

Nomenclature

AUV : Autonomous Underwater Vehicle
DOF : Degree of Freedom
SVD :Singular Value Decompotition
EnKF-SR : Ensemble Kalman Filter Square Root
SNAME : The Society of Naval Architects and Marine Engineers
$\eta = [\eta_1^T, \eta_2^T]^T$: The position and orientation vector in the earth-fixed coordinates
$\eta_1 = [x, y, z]^T$: The linear position vector in the earth-fixed coordinates
$\eta_2 = [\emptyset, \theta, \Psi]^T$: The angular position vector in the earth-fixed coordinates
$v = [v_1^T, v_2^T]^T$: The linear and angular velocity vector in the body-fixed coordinates
$v_1 = [u, v, w]^T$: The linear velocity vector in the body-fixed coordinates
$v_2 = [p, q, r]^T$: The angular velocity vector in the body-fixed coordinates
$\tau = [\tau_1^T, \tau_2^T]^T$: The forces and moments acting on the vehicle in the body-fixed coordinates
$\tau_1 = [X, Y, Z]^T$: The forces acting on the vehicle in the body-fixed coordinates
$\tau_2 = [K, M, N]^T$: The moments acting on the vehicle in the body-fixed coordinates
$[x_G, y_G, z_G]$: The AUV's center of gravity in body fixed coordinates
$[I_x, I_y, I_z]$: The moments of inertia about the X, Y, Z axes respectively
$f_1, f_2, f_3, f_4, f_5, f_6$: Surge,Sway,Heave,Roll,Pitch and Yaw for Function in Jacobian Matrix
$a_1, a_2, \ldots, g_6$: Compoent of Matrix A (Result of linearization using Jacobian Matrix)
$A_1, A_2, \ldots, G_6$: Compoent of Matrix B (Result of linearization using Jacobian Matrix)

I. Introduction

Geographical area of Indonesia consists of islands and waters. Approximately two-thirds of the total area of Indonesia is water. Its strategic position, tropical climate and abundant natural resources offer high economic potential as well as national defensive potential, thus sophisticated underwater robotics technology is required to keep both national security and sea treasure of Indonesia. Underwater robotics technology currently being developed is an Autonomous Underwater Vehicle (AUV). AUV is capably underwater vehicle in moving automatically without direct control by humans according to the trajectory. AUV can be used for underwater exploration, mapping, underwater defense system equipment, sensor off board submarines, inspection of underwater structures, natural resources and the condition of the Earth's surface plates etc [1]. Two important things required to analyze the Autonomous Underwater Vehicle (AUV) that is Earth Fixed Frame (EFF) and Body Fixed Frame (BFF) [2].

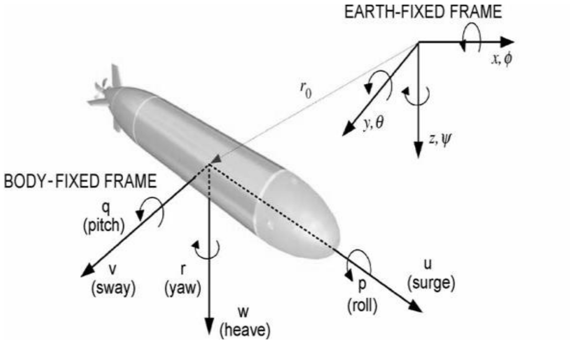

Fig. 1 6 DOF in AUV

In a great number of envioned mission scenarios, AUV will be need to follow inertial reference trajectory accurately [3]. To achieve that purpose, the navigation system must be designed and implemented on AUV. One of basic Navigation system is trajectory estimation was introduced in the 1961s and the most popular of estimation methods is the Kalman Filter. Kalman filter is method of a state variable estimation from linear discrete dynamic system which minimizes the estimation error covariance [4]. Another approximation is an extension of the Kalman filter called Ensemble Kalman Filter (EnKF) and Ensemble Kalman Filter Square Root (EnKF-SR). In the EnKF method, the algorithm is executed by generating a number of specific ensemble to calculate the mean and covariance error state variables [5]. Ensemble Kalman Filter Square Root (EnKF-SR) is development of EnKF algorithm which Square root scheme is one scheme can be implemented in correction step [6].

The main contribution of this paper is trajectory estimation of linear AUV SEGOROGENI ITS system with Ensemble Kalman Filter Square Root (EnKF-SR). Linear model is obtained by linearizing nonlinear 6 DOF AUV model with Jacobian matrix. Linear system is platform to make trajectory estimation. The result of this paper is numeric simulation by comparing real trajectory and the result of trajectory estimation to get a small root mean square error (RMSE). This paper consists of three types of simulations which the first, second and third simulation by generate 100, 200 and 300 ensemble. Profile of SEGOROGENI ITS depicted in Figure 2.

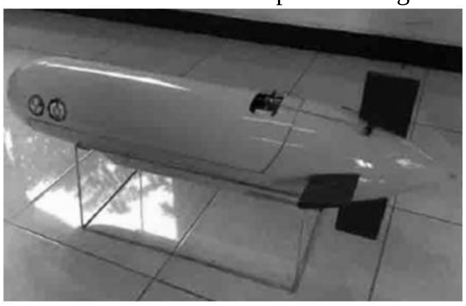

Fig. 2 Profile of AUV SEGOROGENI ITS

II. Mathematical Model

Two important things required to analyze the Autonomous Underwater Vehicle (AUV) that is Earth Fixed Frame (EFF) and Body Fixed Frame (BFF) (Yang,2007). EFF is used to describe the position and orientation of the AUV with the position of the x axis to the north, the y-axis to the east and the z-axis toward the center of the earth while BFF is used to describe the speed and acceleration of the AUV with the starting point at the center of gravity. x-axis to the ship's bow, the positive y axis direct to the right side of the ship and the positive z-axis direct [7,8]. Motion of AUV have 6 DOF where 3 DOF for translational motion and 3 DOF for rotational motion in point x, y and z are listed in Table 1. The dynamics of the AUV there are external forces influencing the movement follows as [9].

$$\tau = \tau_{hidrostatis} + \tau_{addedmass} + \tau_{drag} + \tau_{lift} + \tau_{control} \quad (1)$$

Table 1 Notation of AUV Motion Axis [9]:

DOF	Translational And Rotational	Force / Moment	Linear and Anguler Velocity	Potition/ Angle Euler
1	Surge	X	U	x
2	Sway	Y	V	y
3	Heave	Z	W	z
4	Roll	K	P	ϕ
5	Pitch	M	Q	ϑ
6	Yaw	N	R	ψ

$$\eta = [\eta_1^T, \eta_2^T]^T, \eta_1 = [x, y, z]^T, \quad \eta_2 = [\emptyset, \theta, \Psi]^T ;$$
$$v = [v_1^T, v_2^T]^T, v_1 = [u, v, w]^T, v_2 = [p, q, r]^T$$
$$\tau = [\tau_1^T, \tau_2^T]^T, \tau_1 = [X, Y, Z]^T, \ \tau_2 = [K, M, N]^T \quad (2)$$

Where $\boldsymbol{\eta}$ vector position the position and orientation of the EFF, $\boldsymbol{v}$ vector velocity of linear and anguler of the BFF, the position and orientation of the BFF, and τ description of force and moment in AUV of the BFF. By combining equations hydrostatic, lift, added mass, drag, thrust and assuming a diagonal tensor of inertia (I_o) is zero then the total forces and moments of models obtained from the following [2].

General equation of motion in 6-DOF AUV consists of translational and rotational as follows [2]:

Surge:

$$m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] = X_{|u|u}u|u| + X_{\dot{u}}\dot{u} + X_{wq}wq + X_{qq}qq + X_{vr}vr + X_{rr}rr + X_{prop} \quad (3)$$

Sway :

$$m[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(pq + \dot{r})] = Y_{res} + Y_{|v|v}v|v| + Y_{r|r|}r|r| + Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_{ur}ur + Y_{wp}wp + Y_{pq}pq + Y_{uv}uv + Y_{uu\delta_r}u^2\delta_r \quad (4)$$

Heave :

$$m[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] = Z_{res} + Z_{|w|w}w|w| + Z_{q|q|}q|q| + Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_{uq}uq + Z_{vp}vp + Z_{rp}rp + Z_{uw}uw + Z_{uu\delta_s}u^2\delta_s \quad (5)$$

Roll:

$$I_x\dot{p} + (I_z - I_y)qr + m[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)] = K_{res} + K_{p|p|}p|p| + K_{\dot{p}}\dot{p} + K_{prop} \quad (6)$$

Pitch :

$$I_y\dot{q} + (I_x - I_z)rp + m[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)] = M_{res} + M_{w|w|}w|w| + M_{q|q|}q|q| + M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + M_{uq}uq + M_{vp}vp + M_{rp}rp + M_{uw}uw + M_{uu\delta_s}u^2\delta_s \quad (7)$$

Yaw :

$$I_z\dot{r} + (I_y - I_z)pq + m[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)] = N_{res} + N_{v|v|}v|v| + N_{r|r|}r|r| + N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_{ur}ur + N_{wp}wp + N_{pq}pq + N_{uv}uv + N_{uu\delta_r}u^2\delta_r \quad (8)$$

Translational motion u, v and w are representation of surge, sway and heave. Rotational motion p, q and r are representation of roll, pitch and yaw. This type of AUV SEGOROGENI ITS using only one propeller on the tail AUV which will produce x_{prop} and additional moments K_{prop}. External forces and moments acting on the AUV are the hydrostatic force, thrust and hydrodynamic force and where every object in the water will have a hydrostatic force consisting of gravity and buoyancy forces. While hydrodynamic component consists of added mass, drag and lift. Specification of AUV SEGOROGENI ITS in Table 2.

Table 2. specification of AUV SEGOROGENI ITS

Weight	15 Kg
Overall Length	980 mm
Beam	180 mm
Controller	Ardupilot Mega 2.0
Communication	Wireless Xbee 2.4 GHz
Camera	TTL Camera
Battery	Li-Pro 11,8 V
Propulsion	12V motor DC
Propeller	3 Blades OD : 40 mm
Speed	1,94 knots (1m/s)

III. Linearization

In this paper the nonlinear system of AUV model can be linearized with Jacobian matrix where the nonlinear AUV system in general as follows

$$\dot{x}(t) = f(x(t), u(t), t)$$
$$y(t) = g(x(t), u(t), t) \quad (9)$$

So the Jacobian matrix is formed as follows [10] :

$$\frac{\partial f(\bar{x},\bar{u},t)}{\partial x} = \begin{bmatrix} \frac{\partial f_1(\bar{x},\bar{u},t)}{\partial x_1} & \frac{\partial f_1(\bar{x},\bar{u},t)}{\partial x_2} & \cdots & \frac{\partial f_1(\bar{x},\bar{u},t)}{\partial x_n} \\ \frac{\partial f_2(\bar{x},\bar{u},t)}{\partial x_1} & \frac{\partial f_2(\bar{x},\bar{u},t)}{\partial x_2} & \cdots & \frac{\partial f_2(\bar{x},\bar{u},t)}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_n(\bar{x},\bar{u},t)}{\partial x_1} & \frac{\partial f_n(\bar{x},\bar{u},t)}{\partial x_2} & \cdots & \frac{\partial f_n(\bar{x},\bar{u},t)}{\partial x_n} \end{bmatrix} \quad (10)$$

So equation 3 - 8 can be expressed as follows :

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \frac{mz_G}{m-X_{\dot{u}}} & \frac{-my_G}{m-X_{\dot{u}}} \\ 0 & 1 & 0 & -\frac{mz_G}{m-Y_{\dot{v}}} + & 0 & \frac{(mx_G-Y_{\dot{r}})}{m-Y_{\dot{v}}} \\ 0 & 0 & 1 & \frac{my_G}{m-Z_{\dot{w}}} & -\frac{(mx_G+Z_{\dot{q}})}{m-Z_{\dot{w}}} & 0 \\ 0 & -\frac{m\,z_G}{I_x-K_{\dot{p}}} & \frac{m\,y_G}{I_x-K_{\dot{p}}} & 1 & 0 & 0 \\ \frac{m\,z_G}{I_y-M_{\dot{q}}} & 0 & -\frac{(m\,x_G+M_{\dot{w}})}{I_y-M_{\dot{q}}} & 0 & 1 & 0 \\ -\frac{m\,y_G}{I_z-N_{\dot{r}}} & \frac{(m\,x_G-N_{\dot{v}})}{I_z-N_{\dot{r}}} & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix} \quad (11)$$

Where $f_1, f_2, f_3, f_4, f_5, f_6$ expressed as follows :

$$f_1 = \frac{X_{res} + X_{|u|u}u|u| + X_{wq}wq + X_{qq}qq + X_{vr}vr + X_{rr}rr + X_{prop} - m[-vr + wq - x_G(q^2+r^2) + pq\,y_G + pr\,z_G]}{m - X_{\dot{u}}} \quad (12)$$

$$f_2 = \frac{Y_{res} + Y_{|v|v}v|v| + Y_{r|r|}r|r| + Y_{ur}ur + Y_{wp}wp + Y_{pq}pq + Y_{uv}uv + Y_{uu\delta_r}u^2\delta_r - m[-wp + ur - y_G(r^2+p^2) + qr\,z_G + pq\,x_G]}{m - Y_{\dot{v}}} \quad (13)$$

$$f_3 = \frac{Z_{res} + Z_{|w|w}w|w| + Z_{q|q|}q|q| + Z_{uq}uq + Z_{vp}vp + Z_{rp}rp + Z_{uw}uw + Z_{uu\delta_s}u^2\delta_s - m[-uq + vp - z_G(p^2+q^2) + rp\,x_G + rq\,y_G]}{m - Z_{\dot{w}}} \quad (14)$$

$$f_4 = \frac{K_{res} + K_{p|p|}p|p| + K_{prop} - \left((I_z - I_y)qr + m\begin{bmatrix} y_G(-uq+vp) - \\ z_G(-wp+ur) \end{bmatrix}\right)}{I_x - K_{\dot{p}}} \quad (15)$$

$$f_5 = \frac{M_{res} + M_{w|w|}w|w| + M_{q|q|}q|q| + M_{uq}uq + M_{vp}vp + M_{rp}rp + M_{uw}uw + M_{uu\delta_s}u^2\delta_s - ((I_x - I_z)rp + m[z_G(-vr+wq) - x_G(-uq+vp)])}{I_y - M_{\dot{q}}} \quad (16)$$

$$f_6 = \frac{N_{res} + N_{v|v|}v|v| + N_{r|r|}r|r| + N_{ur}ur + N_{wp}wp + N_{pq}pq + N_{uv}uv + N_{uu\delta_r}u^2\delta_r - ((I_y - I_z)pq + m[x_G(-wp+ur) - y_G(-vr+wq)])}{I_z - N_{\dot{r}}} \quad (17)$$

Furthermore linear system is obtained as follows [11]:

$$\dot{x}(t) = A\,x(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t) \quad (18)$$

with

$A = J_x$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{mz_G}{m-X_{\dot{u}}} & \frac{-my_G}{m-X_{\dot{u}}} \\ 0 & 1 & 0 & -\frac{mz_G}{m-Y_{\dot{v}}} + & 0 & \frac{(mx_G-Y_{\dot{r}})}{m-Y_{\dot{v}}} \\ 0 & 0 & 1 & \frac{my_G}{m-Z_{\dot{w}}} & -\frac{(mx_G+Z_{\dot{q}})}{m-Z_{\dot{w}}} & 0 \\ 0 & -\frac{m\,z_G}{I_x-K_{\dot{p}}} & \frac{m\,y_G}{I_x-K_{\dot{p}}} & 1 & 0 & 0 \\ \frac{m\,z_G}{I_y-M_{\dot{q}}} & 0 & -\frac{(m\,x_G+M_{\dot{w}})}{I_y-M_{\dot{q}}} & 0 & 1 & 0 \\ -\frac{my_G}{I_z-N_{\dot{r}}} & \frac{(mx_G-N_{\dot{v}})}{I_z-N_{\dot{r}}} & 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} a_1 & b_1 & c_1 & d_1 & e_1 & g_1 \\ a_2 & b_2 & c_2 & d_2 & e_2 & g_2 \\ a_3 & b_3 & c_3 & d_3 & e_3 & g_3 \\ a_4 & b_4 & c_4 & d_4 & e_4 & g_4 \\ a_5 & b_5 & c_5 & d_5 & e_5 & g_5 \\ a_6 & b_6 & c_6 & d_6 & e_6 & g_6 \end{bmatrix} \quad (19)$$

$B = J_u$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{mz_G}{m-X_{\dot{u}}} & \frac{-my_G}{m-X_{\dot{u}}} \\ 0 & 1 & 0 & -\frac{mz_G}{m-Y_{\dot{v}}} + & 0 & \frac{(mx_G-Y_{\dot{r}})}{m-Y_{\dot{v}}} \\ 0 & 0 & 1 & \frac{my_G}{m-Z_{\dot{w}}} & -\frac{(mx_G+Z_{\dot{q}})}{m-Z_{\dot{w}}} & 0 \\ 0 & -\frac{m\,z_G}{I_x-K_{\dot{p}}} & \frac{m\,y_G}{I_x-K_{\dot{p}}} & 1 & 0 & 0 \\ \frac{m\,z_G}{I_y-M_{\dot{q}}} & 0 & -\frac{(m\,x_G+M_{\dot{w}})}{I_y-M_{\dot{q}}} & 0 & 1 & 0 \\ -\frac{my_G}{I_z-N_{\dot{r}}} & \frac{(mx_G-N_{\dot{v}})}{I_z-N_{\dot{r}}} & 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} A_1 & B_1 & C_1 & D_1 & E_1 & G_1 \\ A_2 & B_2 & C_2 & D_2 & E_2 & G_2 \\ A_3 & B_3 & C_3 & D_3 & E_3 & G_3 \\ A_4 & B_4 & C_4 & D_4 & E_4 & G_4 \\ A_5 & B_5 & C_5 & D_5 & E_5 & G_5 \\ A_6 & B_6 & C_6 & D_6 & E_6 & G_6 \end{bmatrix} \quad (20)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } D = 0$$

$$\text{So} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = A \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix} + B \begin{bmatrix} X_{prop} \\ \delta_r \\ \delta_s \\ K_{prop} \\ \delta_s \\ \delta_r \end{bmatrix} \quad (22)$$

And then discretized can be show

$$\begin{bmatrix} u_{k+1} \\ v_{k+1} \\ w_{k+1} \\ p_{k+1} \\ q_{k+1} \\ r_{k+1} \end{bmatrix} = A \begin{bmatrix} u_k \\ v_k \\ w_k \\ p_k \\ q_k \\ r_k \end{bmatrix} + B \begin{bmatrix} X_{prop} \\ \delta_r \\ \delta_s \\ K_{prop} \\ \delta_s \\ \delta_r \end{bmatrix} \quad (23)$$

If we write completely, so the discrete models in equation (23) can be written generally in a linear function bellow

$$x_{k+1} = f(x_k, u_k) \quad (24)$$

Due to some assumptions, the stochastic factor in noise must be added to each equations. Thus equation (23) can be formulated as follows [4] :

$$x_{k+1} = f(x_k, u_k) + w_k \quad (25)$$

$$z_k = Hx_k + v_k \quad (26)$$

whereas $f(x_k, u_k)$ is nonlinear or linear function will be applied Ensemble Kalman Filter Square Root (EnKF-SR) Algorithm.

The system noise w_k and measurement noise $v_k$are generated by computer and ussualy normally distributed, so the mean zero. Generally, Q_k states the system noise varians and R_k states the measurement noise varians. Both are depend on time [4].

IV. Ensemble Kalman Filter Square Root

This section present EnKF-SR algorithm to estimated nonlinear or linear dynamic system and measurement model, the algorithm *Ensemble Kalman Filter Square Root* (EnKF-SR) can be seen [6]:

Model system and measurement model

$$x_{k+1} = f(x_k, u_k) + w_k \quad (27)$$

$$z_k = Hx_k + v_k \quad (28)$$

$$w_k \sim N(0, Q_k), \;\; v_k \sim N(0, R_k) \quad (29)$$

1. Initialization

Generate *N ensemble* as the first guess $\bar{x}_0$

$$x_{0,i} = [x_{0,1} \quad x_{0,2} \quad \dots \quad x_{0,N}] \quad (30)$$

The first Mean *Ensemble*: $\bar{x}_{0,i} = x_{0,i}\, 1_N$ (31)

The first *Ensemble* error :

$$\tilde{x}_{0,i} = x_{0,i} - \bar{x}_{0,i} = x_{k,i}(I - 1_N) \quad (32)$$

2. Time Update

$$\hat{x}_{k,i}^- = f(\hat{x}_{k,-1,i}, u_{k-1,i}) + w_{k,i} \quad (33)$$

where $w_{k,i} = N(0, Q_k)$

Mean *Ensemble* :$\bar{x}_{k,i}^- = \hat{x}_{k,i}^-\, 1_N$ (34)

Error *Ensemble* :

$$\tilde{x}_{k,i}^- = \hat{x}_{k,i}^- - \bar{x}_{k,i}^- = \hat{x}_{k,i}^-\,(I - 1_N) \quad (35)$$

3. Measurement Update

$$z_{k,i} = Hx_{k,i} + v_{k,i} \quad (36)$$

where $v_{k,i} \sim N(0, R_k)$

$S_k = H\tilde{x}_{k,i}^-$, $E_k = (v_1, v_2, \dots, V_N)$ and

$$C_k = S_k\, S_k^T + E_k\, E_k^T \quad (37)$$

Mean Ensemble :

$$\bar{x}_{k,i} = \bar{x}_{k,i}^- + \tilde{x}_{k,i}^- S_k^T C_k^- (\bar{z}_{k,i} - H\bar{x}_{k,i}^-) \quad (38)$$

Square Root Scheme:

- eigenvalue decompotition from

$$C_k = U_k\, \Lambda_k U_k^T \quad (39)$$

- determine matrix $M_k = \Lambda_k^{\frac{1}{2}}\, U_k^T\, S_k^-$ (40)

- determine SVD from $M_k = Y_k\, \mathrm{L}_k\, V_k^T$ (41)

Ensemble Error :

$$\tilde{x}_{k,i} = \tilde{x}_{k,i}^-\, V_k\, (I - \mathrm{L}_k\, L_k^T)^{\frac{1}{2}} \quad (42)$$

Ensemble Estimation :

$$\hat{x}_{k,i} = \tilde{x}_{k,i} + \bar{x}_{k,i} \quad (43)$$

To evaluate of estimation result accuracy from EnKF algorithm, can be show with calculate Root Mean Square Error (RMSE) [6].

$$RMSE = \sqrt{\frac{\sum_i^n \left(X_{obs\,,i}(k) - X_{model\;\,,i}(k)\right)^2}{n}} \quad (44)$$

With

$X_{obs,i}(k)$ = observation data

$X_{model\;,i}(k)$ = model data

n = iteration

V. Computational Result

This simulation has been carried out by implementing an algorithm Ensemble Kalman Filter (EnKF) in the AUV model. The result of simulation will be evaluated and compared with real condition, estimaton result with EnKF. This simulation consist of three types of simulations. in which the first, second, third simulation by generate 100,200 and 300 ensemble. Simulations have been done by assuming surge (u), sway (v), heave (w), roll (p), pitch $(q)$and yaw (r). The value of Δt has been used was $\Delta t = 0{,}1$. Initial condition has been used were $u_0 = 0\;m$, $v_0 = 0\;m$,$w_0 = 0\;m$,$p_0 = 0\;rad$, $q_0 = 0\;rad$ and $r_0 = 0\;rad$.

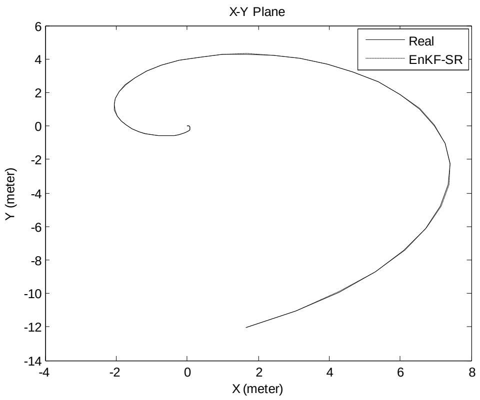

Figure 3. Trajectory Estimation of 6 DOF for XY Plane with 200 ensemble

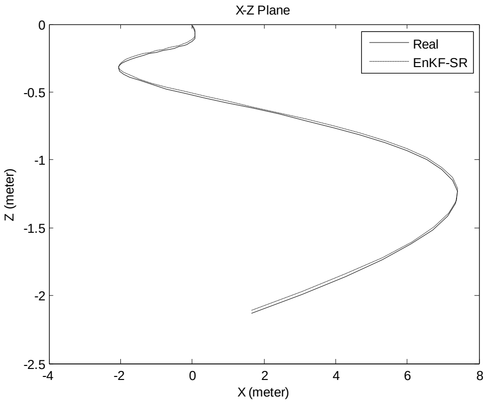

Figure 4. Trajectory Estimation of 6 DOF for XZ Plane with 200 ensemble

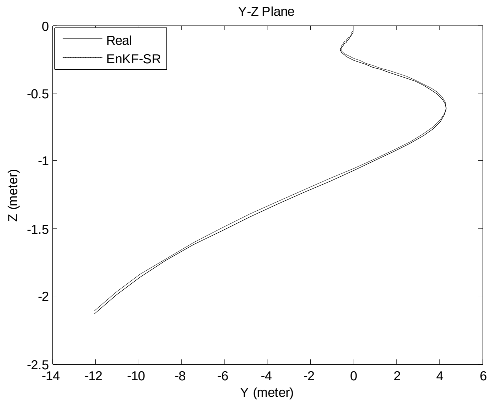

Figure 5. Trajectory Estimation of 6 DOF for YZ Plane with 200 ensemble

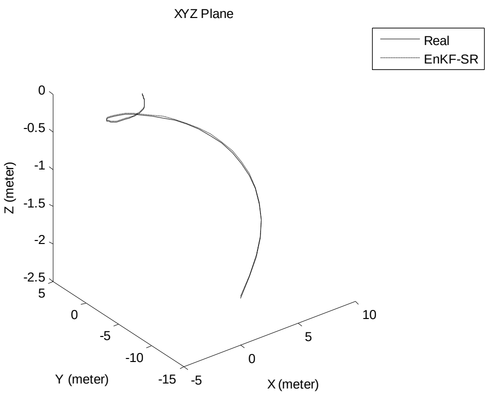

Figure 6. Trajectory Estimation of 6 DOF for XYZ Plane with 200 ensemble

Figure 3 until figure 6 show result of trajectory estimation AUV by generate 200 ensemble. Figure 3 shows the result of trajectory estimation in XY plane, figure 4 in XZ plane, figure 5 in YZ plane and figure 6 in XYZ plane. Figure 3 shows result of trajectory estimation highly accurate with tracking error of surge 0,0094 m/s and accuracy of 99,22%. Tracking error of sway 0,011 m/s and accuracy of 97,8%. AUV moves forward and turns around within XY plane to the right direction reaching more or less 270 degrees. Surge motion is influenced by the propeller X_{prop}. The sway motion is influenced by vertical fin or rudder δ_r. The angle of rudder position will affect sway motion of AUV so we need control system for control the angle of rudder position.

Figure 4 shows result of trajectory estimation highly accurate with tracking error of surge 0,0094 m/s and accuracy of 99,22%. Tracking error of heave 0,0081 m/s and accuracy of 96,8%. AUV dives moving right and left to the depth of 2,2 meters. The heave motion is influenced by horizontal fin or stern δ_s. The angle of stern position will affect heave motion of AUV, so we need control system for control The angle of stern position. Figure 5 shows shows result of trajectory estimation highly accurate for YZ plane. Figure 6 shows result of trajectory estimation highly accurate for XYZ Plane with tracking error of translational motion 0,009 m/s and accuracy of 97,39%. Tracking error of rotational motion 0,001 rad/s and accuracy of 99,97%.

In general as seen in the Table 3, the results of the three simulations were highly accurate. The first simulation by generate 100 ensemble with tracking error of translational motion 0,0093 m/s or accuracy of 97,39% and rotational motion 0,0012 rad/s or accuracy of 99,978%. the second simulation by generate 200 ensemble with tracking error of translational motion 0,0096 m/s or accuracy of 97,94% and rotational motion 0,0015 rad/s or accuracy of 99,96%. The third simulation by generate 300 ensemble with tracking error of translational motion 0,0095 m/s or accuracy of 97,31% and rotational motion 0,0019 rad/s or accuracy of 99,94%. Time simulation of the three simulation results with 100 ensemble faster than 200 and 300 ensemble because more ensemble generated the longer time simulation

Table 3. RMSE value from Computational Result

	100 ens RMSE	Accuracy	200ens RMSE	Accuracy	300ens RMSE	Accuracy
Surge	0.0082 m/s	99,41 %	0.0094 m/s	99,22 %	0.0083 m/s	99,3%
Sway	0.01 m/s	97,77 %	0.0112 m/s	97,8%	0.0101 m/s	97,76 %
Heave	0.0096 m/s	94,98 %	0.0081 m/s	96,8%	0.0108 5 m/s	94,86 %
Roll	0.0012 rad/s	99,99 %	0.0012 rad/s	99,95 %	0.0019 rad/s	99,93 %
Pitch	0.0012 rad/s	99,97 %	0.0017 rad/s	99,96 %	0.0018 rad/s	99,95 %
Yaw	0.001 rad/s	99,97 %	0.0015 rad/s	99,96 %	0.0020 rad/s	99,95 %
Time	0.6875 s		1.0469 s		1.4531 s	

VI. Conclusion

Based on analysis of the three simulation results, EnKF-SR method could be applied to estimate of linear system trajectory of AUV SEGOROGENI ITS with considerably high accuracy. Of the three simulation by generating both 100, 200 and 300 ensembles, the estimation results were all accurate. Square root scheme is one scheme can be implemented in the EnKF. This scheme can affect the estimation results because it implemented in corecction step in EnKF Algorithm.

Reference

[1] Herlambang, T., Nurhadi H and Subchan., 2014a "Preliminary Numerical Study on Designing Navigation and Stability Control Systems for ITS AUV", Applied Mechanics and Materials Vol. 493 (2014) pp 420-425 Trans Tech Publications, Switzerland.

[2] Yang, C. 2007. Modular Modelling and Control for Autonomous Vehicle (AUV). Department of

Mechanical Engineering National University of Singapore
[3] Pascoal, A. (1994).The AUV MARIUS: Mission Scenarios, Vehicle Design, Construction and Testing. Proceedings of the 2nd Workshop on Mobile Robots for Subsea Environments. Monterey Bay Aquarium, Monterey, California, USA.
[4] Lewis, L Frank. (1986), *"Optimal Estimation, With An Introduction To Stochastic Control Theory"*, John Wiley and Sons, New York.
[5] Evensen, G (2009), "*Data Asimilation The Ensemble Kalman Filter (second edition)"*, Springer-Verlag Berlin Hiedelberg London and New York
[6] Herlambang, T. (2012), "Square Root Ensemble Kalman Filter (SR-EnKF) for Estimation of Missile Position", Magister Thesis, Department of Mathematics Sepuluh Nopember Institut of Technology, Surabaya.
[7] Fosen, T. I.2005. A Nonlinear Unified State-space Model for Ship Maneuvering and Control in A Seaway-Journal of Bifurcation and Chaos.
[8] Perez, T. O. N. Smogeli, T.I Fossen and A.J Sorensen. 2005. An Overview of marine Systems Simulator (MSS): A Simulink Toolbox for Narine Control System . SIMS2005-Scandanavian Conference on Simulation and Modelling.
[9] SNAME, The Society of Naval Architects and Marine Engineers, "Nomenclature for Treating the Motion of A Submerged Body Through A Fluid", Technical and Research Bulletin, no. 1-5, 1950.
[10] Subiono. 2010. "mathematical system". Department of Mathematics Sepuluh Nopember Institut of Technology, Surabaya.
[11] Herlambang, T., Djatmiko E.B and Nurhadi H., 2014b. "Optimization with Jacobian Approach for ITS AUV System", international conference on Marine Technology.

Teguh Herlambang was born in 1987, He is currently a Ph.D student at the Department of Ocean EngIneering at Institute of Technology Sepuluh Nopember (ITS) and received his B.Sc and M.Sc. degree from Department of Mathematics at Institute of Technology Sepuluh Nopember (ITS). His area of interest is modelling, navigation, guidance and control of dynamics system.

Eko Budi Djatmiko is currently Professor of Naval Architecture and Ocean Engineering, Institut Teknologi Sepuluh Nopember (ITS), Surabaya, Indonesia. He received the Ph. D (1992) and Master Degree (1988) from the University of Glasgow, UK. Prof. Djatmiko is the dean of the Faculty of Marine Tecnology at ITS. He has published numerous technical papers in his field and successfully supervised more than 10 doctoral students in the subjects of ship and offshore structure hydrodynamics, operability, reliability, as well as surface and underwater vehicle control and guidance.

Hendro Nurhadi Received the Dipl.Ing.(FH) degree from the University of Applied Science Georg-Simon-Ohm Fachhochshule Nuremberg, Nuremberg, Germany, in 2001 and the Ph.D degree from the National Taiwan University of Science and Technology (NTUST), Taipei, Taiwan, in 2009. He is currently with the Department of Mechanical Engineering at Institute of Technology Sepuluh Nopember (ITS), Surabaya, Indonesia, in charged as a Head of Mechatronics Laboratory, also assigned as a coordinator of national consortium for mechatronics for defense, unmanned systems and industrial machineries. He has authored numerous international journal papers and International conferences, as well as reviewer and editor for various international journal papers and international proceedings. His research interests and consulting activities are in the areas of control system, robotics and automation, advanced mechatronics, automated optical inspection (AOI), machine tools, dynamic systems, automation of manufacturing processes, computer-aided design and manufacturing, optimization applications, digital signal processing, artificial intelligence and related fields.

.