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Abstract. A thruster is a device used for station keeping, attitude control, in the reaction control system, or long-duration, low-thrust acceleration. Thruster is one of the main components in autonomous surface vehicle. In this paper, we discuss the linearization of two-state thruster model by using the Jacobian method.

1. Introduction

A thruster is a device used in many equipments including Autonomous Surface Vehicle (ASV) for station keeping, attitude control, in the reaction control system, or long-duration, low-thrust acceleration. The thruster pushes the ASV so that the ASV can move to the desired position. In order to design a controller for thruster, we need the mathematical model of thruster. There are many papers in the literature that discuss the thruster model, such as [1, 2, 3, 4, 5].

In this paper, we focus on the two-state thruster model developed in [3]. The thruster model is a nonlinear ordinary differential equation. Usually, designing a controller for nonlinear systems is not easy task. As such, in this paper, we discuss the linearization of the thruster model by using Jacobian method. In Section 2, we desribe the two-state thruster model in [3]. Then in Section 3, we discuss the linearization of the two-state thruster model. We use two conditions for linearization: tunnel thruster test and open-bladed thruster test. After obtaining the linearized two-state thruster model, we are planning to continue this work to the navigation and stability [6], state estimation [7, 8], control design [9, 10, 11, 12].

2. Two-state Thruster Models

The thruster model in [3] has two state variables, i.e. ω_m and U_a

$$\dot{\omega}_m = f_1(\omega_m, U_a, V_s, U_0) = -K_1\omega_m + K_2 V_s - K_h Q, \tag{1}$$

$$\dot{U}_a = f_2(\omega_m, U_a, V_s, U_0) = -K_4 K_3^{-1}\overline{U_a}|\overline{U_a}| + K_3^{-1} T, \tag{2}$$

where the input variables are V_s and U_0. The expressions for ω_m and U_a are denoted by $f_1(\omega_m, U_a, V_s, U_0)$ and $f_2(\omega_m, U_a, V_s, U_0)$, respectively. In this model, the output variable T is given by

$$T = g(\omega_m, U_a, V_s, U_0) = \mathrm{Lift}(\cos\theta) - \mathrm{Drag}(\sin\theta). \tag{3}$$

The expressions for other variables and parameters in the model are as follows

$$Q = 0.7R(\text{Lift}(\sin\theta) + \text{Drag}(\cos\theta)), \tag{4}$$
$$\overline{U_a} = U_a - U_0, \tag{5}$$
$$\text{Lift} = 0.5\rho V^2 AC_{Lmax}\sin(2\alpha_e), \tag{6}$$
$$\text{Drag} = 0.5\rho V^2 AC_{Dmax}(1-\cos(2\alpha_e)), \tag{7}$$
$$\theta = p - \alpha_e, \tag{8}$$
$$\alpha_e = \left(\frac{\pi}{2} - p\right) - \arctan\left(\frac{U_a}{U_p}\right), \tag{9}$$
$$V^2 = U_p^2 + U_a^2, \tag{10}$$
$$U_p = \frac{0.7R\omega_m}{N}, \tag{11}$$
$$K_3 = \rho AL\gamma, \tag{12}$$
$$K_4 = \rho A\Delta\beta. \tag{13}$$

The interpretation of each variable and parameter in the model is as follows:

Table 1. Variables and parameters in the thruster model.

Q	propeller torque (Nm)	ω_m	motor rotational rate (rad/sec)
N	reduction gear ratio	U_p	propeller velocity (m/s)
R	propeller radius (m)	α_e	effective angle of attack (rad)
p	blade pitch (rad)	U_a	section average flow velocity (m/s)
U_p	propeller velocity (m/s)	ρ	mass density of water (kg/m^3)
A	tunnel cross sectional area (m^2)	Lift	lift force (N)
Drag	drag force (N)	T	thrust force (N)
θ	angle of inlet to blades (rad)	L	tunnel length (m)
γ	effective added mass ratio	$\Delta\beta$	momentum coefficient
U_0	vehicle velocity (m/s)		

3. Linearization of the Thruster Model

In this section, we linearize the state and output equations in the thruster model (1)-(3). The thruster model is linearized around constant solution, i.e. ω_m and U_a are constant functions. Because both ω_m and U_a are constants, we obtain $\dot{\omega}_m = 0$ and $\dot{U}_a = 0$. By substituting $\dot{\omega}_m = 0$ to (1), we have

$$-K_1\omega_m + K_2V_s - K_hQ = 0, \tag{14}$$
$$\omega_m = \frac{K_2V_s - K_hQ}{K_1}. \tag{15}$$

In order to obtain ω_m as a constant function, the input V_s has to be a constant function that can be chosen arbitrarily. Next, $\dot{U}_a = 0$ is substituted to (2), as follows

$$-K_4K_3^{-1}\overline{U_a}|\overline{U_a}| + K_3^{-1}T = 0 \tag{16}$$

$$(U_a - U_0)|U_a - U_0| = \frac{T}{K_4} \tag{17}$$

$$U_a = \begin{cases} U_0 + \sqrt{\frac{T}{K_4}}, & \text{if } \frac{T}{K_4} \geq 0, \\ U_0 - \sqrt{\frac{T}{K_4}}, & \text{if } \frac{T}{K_4} < 0. \end{cases} \tag{18}$$

Since U_a is a constant function, the input U_0 must be a constant function which can be chosen freely. Then we determine the the Jacobian matrix of the system (1)-(3). As a first step, we determine the partial derivative of f_1, f_2 and g w.r.t. state variable ω_m. More precisely, we compute $\frac{\partial f_1}{\partial \omega_m}$, $\frac{\partial f_2}{\partial \omega_m}$ and $\frac{\partial g}{\partial \omega_m}$:

$$\frac{\partial f_1}{\partial \omega_m} = -K_1 - K_h \frac{\partial Q}{\partial \omega_m}, \tag{19}$$

$$\frac{\partial f_2}{\partial \omega_m} = \frac{1}{K_3} \frac{\partial g}{\partial \omega_m}, \tag{20}$$

$$\frac{\partial g}{\partial \omega_m} = \cos\theta \frac{\partial \text{Lift}}{\partial \omega_m} - \text{Lift}(\sin\theta) \frac{\partial \theta}{\partial \omega_m} - \sin\theta \frac{\partial \text{Drag}}{\partial \omega_m} - \text{Drag}(\cos\theta) \frac{\partial \theta}{\partial \omega_m}, \tag{21}$$

$$\frac{\partial Q}{\partial \omega_m} = 0.7R \left(\sin\theta \frac{\partial \text{Lift}}{\partial \omega_m} + \text{Lift}(\cos\theta) \frac{\partial \theta}{\partial \omega_m} + \cos\theta \frac{\partial \text{Drag}}{\partial \omega_m} - \text{Drag}(\sin\theta) \frac{\partial \theta}{\partial \omega_m} \right), \tag{22}$$

$$\frac{\partial \text{Lift}}{\partial \omega_m} = 0.5\rho A C_{Lmax} \left(\sin(2\alpha_e) \frac{\partial}{\partial \omega_m}(V^2) + 2V^2 \cos(2\alpha_e) \frac{\partial \alpha_e}{\partial \omega_m} \right), \tag{23}$$

$$\frac{\partial \text{Drag}}{\partial \omega_m} = 0.5\rho A C_{Dmax} \left((1 - \cos(2\alpha_e)) \frac{\partial}{\partial \omega_m}(V^2) + 2V^2 \sin(2\alpha_e) \frac{\partial \alpha_e}{\partial \omega_m} \right), \tag{24}$$

$$\frac{\partial \theta}{\partial \omega_m} = -\frac{\partial \alpha_e}{\partial \omega_m}, \tag{25}$$

$$\frac{\partial \alpha_e}{\partial \omega_m} = \frac{U_a}{U_p^2 + U_a^2} \frac{\partial U_p}{\partial \omega_m}, \tag{26}$$

$$\frac{\partial}{\partial \omega_m}(V^2) = 2U_p \frac{\partial U_p}{\partial \omega_m}, \tag{27}$$

$$\frac{\partial U_p}{\partial \omega_m} = \frac{0.7R}{N}. \tag{28}$$

Then, we compute the partial derivative of f_1, f_2 and g w.r.t. state variable U_a. More specifically, we determine $\frac{\partial f_1}{\partial U_a}$, $\frac{\partial f_2}{\partial U_a}$ and $\frac{\partial g}{\partial U_a}$:

$$\frac{\partial f_1}{\partial U_a} = -K_h \frac{\partial Q}{\partial U_a}, \tag{29}$$

$$\frac{\partial f_2}{\partial U_a} = -2K_4 K_3^{-1} |\overline{U_a}| + K_3^{-1} \frac{\partial g}{\partial U_a}, \tag{30}$$

$$\frac{\partial g}{\partial U_a} = \cos\theta \frac{\partial \text{Lift}}{\partial U_a} - \text{Lift}(\sin\theta) \frac{\partial \theta}{\partial U_a} - \sin\theta \frac{\partial \text{Drag}}{\partial U_a} - \text{Drag}(\cos\theta) \frac{\partial \theta}{\partial U_a}, \tag{31}$$

$$\frac{\partial Q}{\partial U_a} = 0.7R \left(\sin\theta \frac{\partial \text{Lift}}{\partial U_a} + \text{Lift}(\cos\theta) \frac{\partial \theta}{\partial U_a} + \cos\theta \frac{\partial \text{Drag}}{\partial U_a} - \text{Drag}(\sin\theta) \frac{\partial \theta}{\partial U_a} \right), \tag{32}$$

$$\frac{\partial \text{Lift}}{\partial U_a} = 0.5\rho A C_{Lmax} \left(\sin(2\alpha_e) \frac{\partial}{\partial U_a}(V^2) + 2V^2 \cos(2\alpha_e) \frac{\partial \alpha_e}{\partial U_a} \right), \tag{33}$$

$$\frac{\partial \text{Drag}}{\partial U_a} = 0.5\rho A C_{Dmax} \left((1 - \cos(2\alpha_e)) \frac{\partial}{\partial U_a}(V^2) + 2V^2 \sin(2\alpha_e) \frac{\partial \alpha_e}{\partial U_a} \right), \tag{34}$$

$$\frac{\partial \theta}{\partial U_a} = -\frac{\partial \alpha_e}{\partial U_a}, \tag{35}$$

$$\frac{\partial \alpha_e}{\partial U_a} = -\frac{U_p}{U_p^2 + U_a^2}, \tag{36}$$

$$\frac{\partial}{\partial U_a}(V^2) = 2U_a, \tag{37}$$

$$\frac{\partial U_p}{\partial U_a} = 0. \tag{38}$$

Next, we calculate the partial derivative of f_1, f_2 and g w.r.t. input variable V_s, i.e. we compute $\frac{\partial f_1}{\partial V_s}$, $\frac{\partial f_2}{\partial V_s}$ and $\frac{\partial g}{\partial V_s}$:

$$\frac{\partial f_1}{\partial V_s} = K_2, \qquad \frac{\partial f_2}{\partial V_s} = 0, \qquad \frac{\partial g}{\partial V_s} = 0. \tag{39}$$

Finally, we determine the partial derivative of f_1, f_2 and g w.r.t. input variable U_0. More precisely, we calculate $\frac{\partial f_1}{\partial U_0}$, $\frac{\partial f_2}{\partial U_0}$ and $\frac{\partial g}{\partial U_0}$:

$$\frac{\partial f_1}{\partial U_0} = 0, \qquad \frac{\partial f_2}{\partial U_0} = 2K_4K_3^{-1}|\overline{U_a}|, \qquad \frac{\partial g}{\partial U_0} = 0. \tag{40}$$

3.1. Tunnel Thruster Test

In this subsection, we determine the linear system obtained by using the parameters used in the tunnel thruster test [3]. The parameters are shown in Table 2.

Table 2. Parameters for the tunnel thruster test.

$C_{Lmax} = 1.75$	$K_1 = 70.15$	$A = 0.00445$ m^2
$C_{Dmax} = 1.2$	$K_2 = 1133.2$	$D = 0.0762$ m
$\Delta\beta = 0.2$	$K_h = 17.790$	$R = D/2$ m
$\gamma = 0.5$	$K_3 = 0.954$	$L = 0.4191$ m
	$K_4 = 0.910$	$N = 2$
		$\rho = 998$ kg/m^3
		$p = \pi/6$ rad

After substituting the parameters to the linearized system, we obtain the following linear system:

$$\begin{bmatrix} \dot{\omega}_m \\ \dot{U}_a \end{bmatrix} = \begin{bmatrix} -70.7 & 1.2 \\ -2.1 & -14.8 \end{bmatrix} \begin{bmatrix} \omega_m \\ U_a \end{bmatrix} + \begin{bmatrix} 1133.2 & 0 \\ 0 & 1.9 \end{bmatrix} \begin{bmatrix} V_s \\ U_0 \end{bmatrix}, \tag{41}$$

$$T = \begin{bmatrix} -2.03 & -12.2 \end{bmatrix} \begin{bmatrix} \omega_m \\ U_a \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} V_s \\ U_0 \end{bmatrix} \tag{42}$$

3.2. Open-Bladed Thruster Test

In this subsection, we determine the linear system obtained by using the parameters used in the open-bladed thruster test [3]. The parameters are displayed in Table 3.

Table 3. Parameters for the open-bladed thruster test.

$C_{Lmax} = 2$	$K_1 = 10.8$	$A = 0.00445$ m^2
$C_{Dmax} = 0.5$	$K_2 = 0.65$	$D = 0.15$ m
$\Delta\beta = 1.7$	$K_h = 8333$	$R = D/2$ m
$\gamma = 2.26$	$K_3 = 4.0$	$L = 0.10$ m
	$K_4 = 30.0$	$N = 2$
		$\rho = 998$ kg/m^3
		$p = \pi/4$ rad

Then we substitute the parameters to the linearized system. We obtain the following linear system:

$$\begin{bmatrix} \dot{\omega}_m \\ \dot{U}_a \end{bmatrix} = \begin{bmatrix} -523.7 & -1519.8 \\ -1.15 & -0.13 \end{bmatrix} \begin{bmatrix} \omega_m \\ U_a \end{bmatrix} + \begin{bmatrix} 0.65 & 0 \\ 0 & 2.73 \end{bmatrix} \begin{bmatrix} V_s \\ U_0 \end{bmatrix}, \tag{43}$$

$$T = \begin{bmatrix} -4.62 & 10.4 \end{bmatrix} \begin{bmatrix} \omega_m \\ U_a \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} V_s \\ U_0 \end{bmatrix} \tag{44}$$

4. Conclusions

In this paper, we have linearized the two-state thruster model by using the Jacobian method. We are planning to extend this work to navigation, stability, trajectory estimation and control design of the linearized model.

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