Optimal Control Model of Two Dimensional Missile Using Forward Backward Sweep Method (FBSM)

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***Abstract*—Indonesia is an archipelagic and maritime country, so it is imperative to improve the country's aerospace technology, that is, the main equipment of defence system to defend the state sovereignty. One example is a missile that can be remotely controlled. Missiles are military rocket weapons with automatic control system to trace targets or follow direction. One of the missile technologies currently being developed is the optimal control of missile. The application of optimal control by Forward Backward Sweep Method (FBSM) method can be used for missile model consisting of flight angle, speed, horizontal position, and altitude with thrust force as control. FBSM uses state variable and adjoint variable in its computation. Then, FBSM updates the control by current control and new control. Based on the simulation results, the comparison between the missile model with thrust force control and without thrust force control are obtained. The flight angle with control produces smaller deviation than the flight angle without control. The altitude with control produces increasing trajectory while the altitude without control produces decreasing trajectory.**

Keywords— Missile, Optimal Control, Forward Backward Sweep Method

I. Introduction

Indonesia is an archipelagic and maritime country, so it is imperative to improve the country aerospace technology, that is, the main equipment of the state defence system to keep the state sovereignty. One example is a missile that can be remotely controlled. Missiles are military rocket weapons having an automatic control system to trace the target or to follow the direction [15,16]. In this research, the object used is two dimensional missile consisting of flight angle, speed, horizontal position, and altitude with thrust force as control. One of the technologies currently developed is optimal control of missile. The optimal control of missile is used for minimizing flight angle and thrust force.

Optimal control applications have been widely used such as the simple inverted pendulum model with pendulum and cart [4,18], the autonomous underwater vehicle model with six degrees of freedom [9], robotics and mobile robot in estimating trajectories [11,17], arm model consisting of shoulder joint angle and elbow joint angle [12], steam drum boiler in estimating water level and steam temperature [13] and the optimal control for disease spread model of dengue [6], influenza [7], cancer [8]. An optimal control application can also be used for a missile model. The variables used for the optimal control of missile are angle, speed, horizontal position, and altitude with thrust force as control [14].

From the some researches, the Kalman Filter method for estimation has been applied for the missile model [10]. In this research, the optimal missile control model is constructed. There are several methods to solve the optimal control problems such as the Linear Quadratic Regulator (LQR) [2], the Linear Quareatic Tracking (LQT) that is the development of LQR [3], the Forward Backward Difference [1], the Proportional-Integral-Derivative (PID) control for analyzing the response of control [5]. Since the missile model is a nonlinear model, then the method used to solve the optimal system problems and to offer numerical solution is the Forward Backward Sweep Method (FBSM). In the previous research FBSM is used for the disease spread problem [6].

For FBSM, the variables used are state variable with initial condition and adjoint variable with final time condition for their iterative calculations [1]. Then, FBSM updates the control by current control and new control. Based on the simulation results, the comparison between the missile with the thrust force control and without thrust force are obtained. The flight angle with control produced smaller deviation value than that without control. The flight altitude with control produced an increasing trajectory compared to that without control which produced an decreasing trajectory.

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II. MATHEMATICAL MODEL OF TWO DIMENSIONAL MISSILE

The dynamic system of the two-dimensional missile is mathematically modelled based on physical state. The following is the geometrical interpretation of the two-dimensional missile system [10].

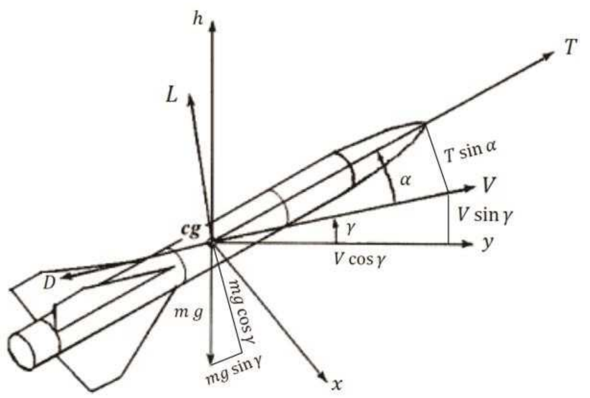

Fig. 1. Dynamic Model of Two Dimensional Missile

Based on Fig. 1 the physical state of the missile is modelled to form the following system of dynamic equations :
The forces working on the plane of rotation are:

$$\tau = I\varphi = mv\frac{d\gamma}{dt} \quad (1)$$

$$\frac{d\gamma}{dt} = \frac{\tau}{mv} \quad (2)$$

Moment τ working on the coordinate system of the missile based Newton Law:

$$\tau = T\sin\alpha + L - mg\cos\gamma \quad (3)$$

Substitute (2) in (3) so that we obtain change in angular velocity of the missile is in (4)

$$\frac{d\gamma}{dt} = \frac{1}{mv}\left(T\sin\alpha + L\right) - \frac{g\cos\gamma}{v} \quad (4)$$

Then, the force F working on translational plane is:

$$F = ma = m\frac{dv}{dt} \quad (5)$$

When m is the mass, a is the acceleration, and v is the speed of the object, the following is obtained.

$$\frac{dv}{dt} = \frac{F}{m} \quad (6)$$

The force working on the coordinate system of the missile based Newton Law is:

$$F = T\cos\alpha - D - mg\sin\gamma \quad (7)$$

So that the change in the speed is obtained as follows:

$$\frac{dv}{dt} = \frac{1}{m}\left(T\cos\alpha - D\right) - g\sin\gamma \quad (8)$$

When T is the force thrust, D is drag, L is lift, α is attack angle, and γ flight angle.
The change in horizontal position of the missile is :

$$\frac{dx}{dt} = V\cos\gamma \quad (9)$$

While, the change in altitude of the missile is:

$$\frac{dh}{dt} = V\sin\gamma \quad (10)$$

So, the form of the optimal control of the missile motion is:

$$\dot{\gamma} = \frac{T}{mv}\sin\alpha + \frac{L}{mv} - \frac{g\cos\gamma}{v} \quad (11)$$

$$v = \frac{T}{m}\cos\alpha - \frac{D}{m} - g\sin\gamma \quad (12)$$

$$\dot{x} = v\cos\gamma \quad (13)$$

$$\dot{h} = v\sin\gamma \quad (14)$$

It is assumed that the attack angle α is very small, so that $\sin\alpha \approx \alpha, \cos\alpha \approx 1$.
In the model, there are the aerodynamic force consisting of axial aerodynamic force and normal aerodynamic force. The axial aerodynamic force is drag D and the normal aerodynamic force is the lift L with the equations are in (15) – (19). [10]

$$D(h,v,\alpha) = \frac{1}{2}C_d\rho v^2 S_{ref} \quad (15)$$

$$C_d = A_1\alpha^2 + A_2\alpha + A_3 \quad (16)$$

$$L(h,v,\alpha) = \frac{1}{2}C_l\rho v^2 S_{ref} \quad (17)$$

$$C_l = B_1\alpha + B_2 \quad (18)$$

$$\rho = C_1h^2 + C_2h + C_3 \quad (19)$$

when ρ is air density, S_{ref} is an area used by the missile.

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In the missile model, there is a thrust control T applied to the state equation [10]. The performance index used in the optimal control is:

$$\min J(T) = \int_0^{T} \left(W_1 \gamma(t)^2 + W_2 T(t)^2 \right) dt \qquad (20)$$

With the weight of $W_1 > 0, W_2 > 0$ related to the flight angle and the thrust force respectively. In the optimal missile control, the flight angle and the thrust force are minimalized. The aim of the optimal control problem is to find out the value of T^* so as to minimize the performance index $J(T^*) = \min(J(T))$.

III. Hamiltonian Form

If T^* is the optimal control of the state system, then there is a adjoint (co-state) variable, that is :

$$(\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4)$$

that meets the following equation system as follows:

$$\frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial \gamma} \qquad (21)$$

$$\frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial v} \qquad (22)$$

$$\frac{d\lambda_3}{dt} = -\frac{\partial H}{\partial x} \qquad (23)$$

$$\frac{d\lambda_4}{dt} = -\frac{\partial H}{\partial h} \qquad (24)$$

With final value $\lambda_1(T) = \lambda_2(T) = \lambda_3(T) = \lambda_4(T) = 0$. Hamiltonian form is as in (26)

$$\begin{aligned} H = {} & W_1 \gamma(t)^2 + W_2 T(t)^2 + \\ & \lambda_1 \left(\frac{T}{mv} \sin\alpha + \frac{L}{mv} - \frac{g\cos\gamma}{v} \right) + \\ & \lambda_2 \left(\frac{T}{m} \cos\alpha - \frac{D}{m} - g\sin\gamma \right) + \\ & \lambda_3 (v\cos\gamma) + \lambda_4 (v\sin\gamma) \end{aligned} \qquad (25)$$

Then the optimal control can be calculated in (28).

$$\frac{\partial H}{\partial T} = 0 \qquad (26)$$

$$2W_2 T + \lambda_1 \left(\frac{\sin\alpha}{mv} \right) + \lambda_2 \left(\frac{\cos\alpha}{m} \right) = 0 \qquad (27)$$

$$T = \frac{\dfrac{-\lambda_1 \sin\alpha - \lambda_2 v \cos\alpha}{mv}}{2W_2} \qquad (28)$$

IV. Forward Backward Sweep Method

Given the initial condition variable x and the final time condition for adjoint variable, the steps for using the Forward Backward Sweep Method (FBSM) areas follows [1]:

1. Set the initial guess for the control u over the interval.
2. Use the initial condition of the state variabel $x(t_0) = a$ and the value for control u, set the solution to the state variabel x forward using Runge Kutta algorithm.
3. Use the final time condition $\lambda(T) = 0$ and the value for control u and the state variable x, determine the solution tothe adjoint variable λ backward using Runge Kutta algorithm.
4. Update the control u based on the new state variable x and the new adjoint variable λ.
5. Repeat the step 1-4 until the process converges.
6. Calculate the performance index in (20).

For the optimal missile control, the steps for applying FBSM are as follows :

Supposing the state variable and the adjoint variable are as follows :

$$f_1 = \frac{d\gamma}{dt}, f_2 = \frac{dv}{dt}, f_3 = \frac{dx}{dt}, f_4 = \frac{dh}{dt}$$

$$g_1 = \frac{d\lambda_1}{dt}, g_2 = \frac{d\lambda_2}{dt}, g_3 = \frac{d\lambda_3}{dt}, g_4 = \frac{d\lambda_4}{dt}$$

With the parameter of the performance index $W_1 > 0, W_2 > 0$

1. Calculate the solution to the state variable forward with the initial condition $X(0) = (\gamma(0), v(0), x(0), h(0))$ using Runge Kutta algorithm

$$k_{1i} = f_i(t, X_i(t), T(t)), i = 1,2,3,4$$

$$k_{2i} = f_i\left(t + \frac{d}{2}, X_i(t) + \frac{d}{2}k_{1i}, \frac{T(t) + T(t+d)}{2}\right), i = 1,2,3,4$$

$$k_{3i} = f_i\left(t + \frac{d}{2}, X_i(t) + \frac{d}{2}k_{2i}, \frac{T(t) + T(t+d)}{2}\right), i = 1,2,3,4$$

$$k_{4i} = f_i\left(t + d, X_i(t) + dk_{3i}, T(t+d)\right), i = 1,2,3,4$$

$$X_i(t+d) = x_i(t) + \frac{d}{6}\left(k_{1i} + 2k_{2i} + 2k_{3i} + k_{4i}\right), i = 1,2,3,4$$

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2. Calculate the solution to the adjoint variable backward with the final condition $\lambda(T)=(\lambda_1(T),\lambda_2(T),\lambda_3(T),\lambda_4(T))$ using Runge Kutta algorithm

$$l_{1i}=g_i(t,\lambda_i(t),X_i(t),T(t)),i=1,2,3,4$$

$$l_{2i}=g_i\left(t-\frac{d}{2},\lambda_i(t)-\frac{d}{2}l_{1i},\frac{X(t)+X(t-d)}{2},\frac{T(t)+T(t-d)}{2}\right)$$
$$i=1,2,3,4$$

$$l_{3i}=g_i\left(t-\frac{d}{2},\lambda_i(t)-\frac{d}{2}l_{2i},\frac{X(t)+X(t-d)}{2},\frac{T(t)+T(t-d)}{2}\right)$$
$$i=1,2,3,4$$

$$l_{4i}=g_i\left(t-d,\lambda_i(t)-dl_{2i},X(t-d),T(t-d)\right),i=1,2,3,4$$

$$\lambda_i(t-d)=\lambda_i(t)-\frac{d}{6}(l_{1i}+2l_{2i}+2l_{3i}+l_{4i}),i=1,2,3,4$$

3. Calculate the optimal control T^* using equation (28).
4. Update the optimal control using equation (29)

$$T\leftarrow\frac{T+T_{old}}{2} \quad (29)$$

5. Repeat the steps until convergent

V. Results

In the missile simulation, the parameter used can be seen in Table 1 [10]. The initial value used was :

$$\gamma(0)=1, v(0)=2, x(0)=4, h(0)=5.$$

With $\gamma(t)$ in degree, $v(t)$ in m/s, $x(t)$ in meter, and $h(t)$ in meter. Whereas, for the parameter of FBSM, the performance index was $W_1=2, W_2=10^{-10}$.

Fig. 2 shows the comparison between the numeric solution to the flight angle with thrust force control and without thrust force control. For the model with control, the resulted flight angle resulted is approaching to 0 deg. While for the model without control, it shows that the flight angle has a higher deviation approaching to -1.5 deg after long time. Thus, the flight angle with control has less square of deviation value than that without control.

Fig. 3 shows the comparison between the numeric solution to the speed with thrust force control and without thrust force control. The speed with control produced higher value and fluctuative than that without control. In the speed with control, in the early time until *t=2.1* second, the speed increases and boosts until 1.2917×10^3 m/s. In the time *t=2.2* until *t=18.6* second, the speed decreases until 560.9587 m/s. In the time *t=18.7* until *t=35.8* second, the speed increases again until 1.2171×10^3 m/s. In the time *t=35.9* until maximum time, the speed decreases until 554.37 m/s.

TABLE I. Parameter for Missile Dynamical System

Parameter	Value	Unit
M	1000	Kg
G	10	m/s^2
S_{ref}	0.3376	m^2
A_1	-1.9431	
A_2	-0.1499	
A_3	0.2359	
B_1	21.9	
B_2	0	
α	0	
C_1	3.312×10^{-9}	$\frac{kg^2}{m}$
C_2	1.142×10^{-4}	$\frac{kg^2}{m}$
C_3	1.224	$\frac{kg^2}{m}$

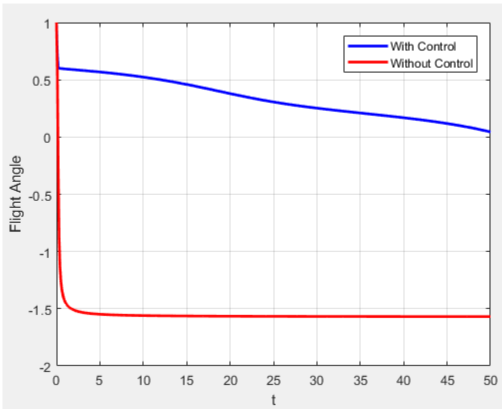

Fig. 2. Numeric solution to the flight angle

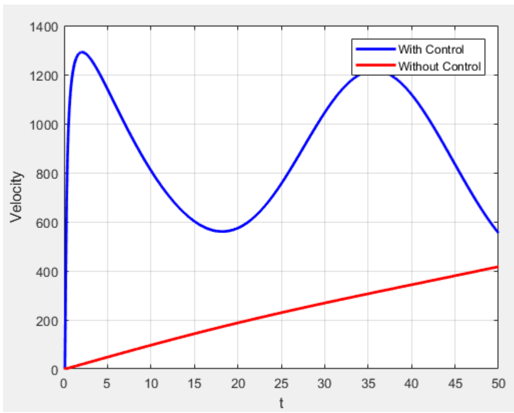

Fig. 3. Numeric solution to velocity

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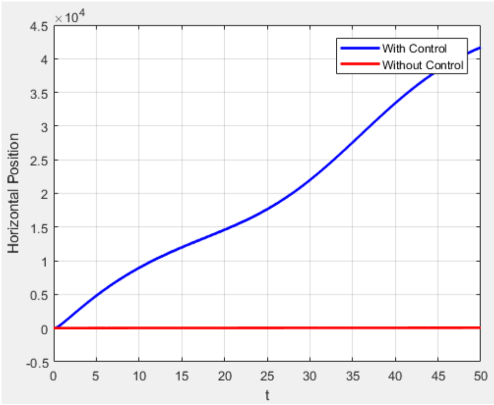


Fig. 4. Numeric solution to horizontal position

Fig. 4 shows the comparison between the numeric solution to horizontal position with thrust force control and without thrust force control. The horizontal position with control has higher value than that without control. Horizontal position with control results length is 41695 meters in maximum time.

Fig. 5 shows the altitude with thrust force control and without thrust force control. For the altitude with control, the altitude of missile increases along iteration. While, in the altitude of missile without control, the missile falls down. Altitudee with control results height is 14255,86 meters in maximum time. So, the altitude with control has upward trend in value, while that without control has downward trend in value.

Fig. 6(a) and Fig. 6(b) shows the optimal control in thrust force. It seems that the thrust boosts and increases from early time until *t=0.2* second with maximum thrust force is 1915133 Newton and then decreases to stable until maximum time.

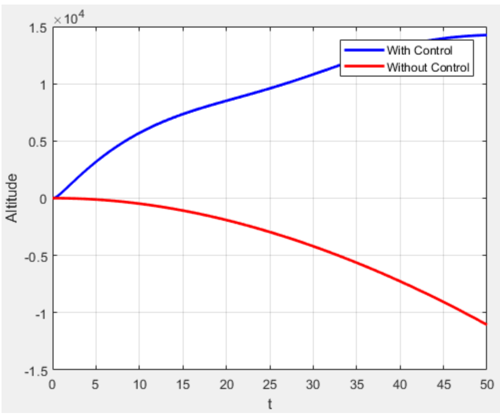
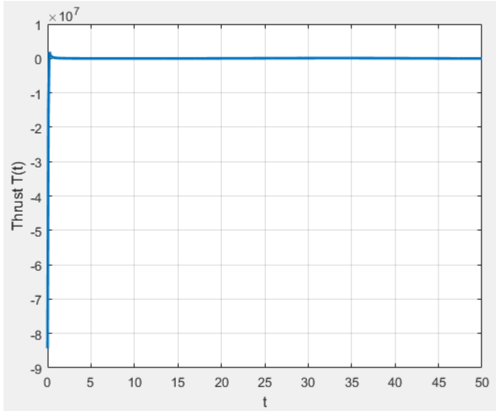
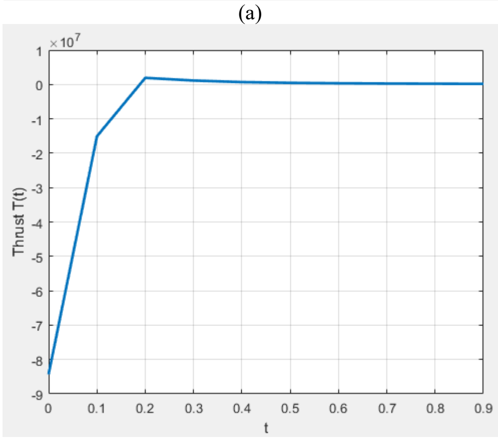


Fig. 5. Numeric solution to altitude



(a)



(b)

Fig. 6. Optimal Control in thrust (a) In whole time (b) In magnifying time 0 to 0.9

VI. Conclusion

The optimal control application by FBSM is applicable to the optimal missile control consisting of flight angle, speed, horizontal position, and and altitude with trust force as control. FBSM uses state variable and adjoint variable in their computation. Then, FBSM updates the control by current control and new control. Based on the simulation results, the comparison between the missile model with thrust force control and without thrust force control are obtained. The flight angle with control produces smaller deviation than the flight angle without control. The altitude with control produces increasing trajectory while the altitude without control produces decreasing trajectory. Developments of this research are building missile model with the more degree of freedom.

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References

[1] S. Lenhart, J.T. Workman., Optimal Control Applied to Biological Models, Chapman & Hall/CRC, New York, 2007

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[2] F.L. Lewis, D.L. Vrabie, V.L. Syrmos., Optimal Control, John Wiley and Sons, New Jersey, 2012

[3] D.S. Naidu, Optimal Control System, CRC Press, Florida, 2003

[4] K. Ogata, Modern Control Engineering, Prentice Hall, New Jersey, 2002

[5] T. Herlambang, D. Rahmalia, T. Yulianto "Particle Swarm Optimization (PSO) and Ant Colony Optimization (ACO) for Optimizing PID Parameters on Autonomous Underwater Vehicle (AUV) Control System," in *Journal of Physics : Conference Series 1211*, 2019, pp. 012039.

[6] D. Rahmalia, T. Herlambang, A.M. Rohmah, A. Muhith "Weights Optimization Using Firefly Algorithm on Optimal Control of Zika Disease from Dengue Symptoms by Vaccination," in *Journal of Physics : Conference Series 1594*, 2020, pp. 012040.

[7] D. Rahmalia, T. Herlambang, "Weight Optimization of Optimal Control Influenza Model Using Artificial Bee Colony," *International Journal of Computing Science and Applied Mathematics*, vol. 4, no. 1, pp. 27-31, 2018.

[8] A. Muhith, D. Rahmalia, T. Herlambang "The Effects of Drugs in Chemotherapy as Optimal Control of Tumor Growth Dynamical Model," in *Journal of Physics : Conference Series 1663*, 2020, pp. 012006.

[9] T. Herlambang, D. Rahmalia, H. Nurhadi, D. Adzkiya, S. Subchan "Optimization of Linear Quadratic Regulator with Tracking Applied to Autonomous Underwater Vehicle (AUV) Using Cuckoo Search" *Nonlinear Dynamics and Systems Theory*, vol. 20, no. 3, pp. 282-298, 2020.

[10] T. Herlambang, "Design of a Navigation and Guidance System of Missile with Trajectory Estimation Using Ensemble Kalman Filter Square Root (EnKF-SR). International Conference on Computer Applications and Information Processing Technology (CAIPT)-IEEE, Bali Indonesia 8-10 Augsut (2017).

[11] T. Herlambang, D. Rahmalia, A. Muhith, H. Nurhadi "Motion Optimization Using Modified Kalman Filter for Invers Kinematics Based Multi DOF Arm Robot" *International Journal of Control and Automation*, vol. 13, no. 2, pp. 64-71, 2020.

[12] D. Rahmalia, T. Herlambang, S. Pancahayani, K. Mu'tamar "Optimization of Linear Quadratic Tracking (LQT) Weight Matrices Using Simulated Annealing Applied on Planar Arm Model" *SPECTA Journal of Technology*, vol. 4, no. 3, pp. 22-35, 2020.

[13] T. Herlambang, Z. Mufarrikoh, D.F. Karya, D. Rahmalia "Estimation of Water Level and Steam Temperature Using Ensemble Kalman Filter Square Root (EnKF-SR)," in *Journal of Physics : Conference Series 1008*, 2018, pp. 012026.

[14] S. Subchan, "A Direct Multiple Shooting Method for Missile Trajectory Optimization with the Terminal Bunt Manoeuvre," *IPTEK The Journal for Technology and Science*, vol. 22, no. 3, pp. 147-151, 2011.

[15] R. Effendie, M.R. Prasetyo, Z. Hidayat, "Designing an Optimal Control LQT for Controlling and Guidance of Missile," *JAREE Journal on Advanced Research in Eletrical Engineering*, vol. 2, no. 1, pp. 24-30, 2018.

[16] G.M. Siouris, Missile Guidance and Control System, Springer, New York, 2010

[17] T. Herlambang, R.A. Rasyid, S. Hartatik, D. Rahmalia, "Estimasi Posisi Mobile Robot Menggunakan Metode Akar Kuadrat Unscented Kalman Filter," *Technology, Science and Engineering Journal*, vol. 1, no. 2, pp. 140-146, 2017.

[18] D. Rahmalia, "Optimisasi Matriks Bobot pada Linear Quadratic Regulator (LQR) Inverted Pendulum Menggunakan Ant Colony Optimization," *MathVision*, vol. 1, no. 2, pp. 61-66, 2019.

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