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*By* Teguh Herlambang & Subchan

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# Missile Position Estimation Using Unscented Kalman Filter

**Teguh Herlambang** [1*], **Subchan**[2]

*[1]Information System Department, FEBTD, Universitas Nahdlatul Ulama Surabaya*
*[2]Mathemathics Department FSAD, Institut Tekologi Sepuluh Nopember*

*Corresponding author e-mail: [1*] teguh@unusa.ac.id*

**Abstract.** *Missiles are military rocket weapons having an automatic control system to locate its targets or adjust its direction. Indonesia itself, which is a country of archipelago, covers air area of its largest territory, followed by sea area and land area. Logically, the existence of missile defense equipment (the main weapon system) or precisely the type of long-range missile is acceptable to support the defense and security of the Republic of Indonesia, but its consequences to be operated in the territory of Indonesia itself, in case of an occufanct of an error in targeting the target, will fall on of harm to its own national territory. Therefore, trajectory estimation for guided missiles is the basic requirement for guided missiles to be aimed at the precise targets. The trajectory is used as a guide to direct that the missile reach the target by following the given path. To maintain the accuracy of the trajectory continuously, the missile trajectory estimation was made by using Unscented Kalman Filter (UKF) Algorithm. This algorithm was used to estimate nonlinear dynamic models The simulation results showed that the UKF method was effective, showing the accuracy of 97% by the UKF method*

***Keywords:*** *Missile,, UKF, Trajectory, Estimation, Position, Kalman Filter*

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## 1. INTRODUCTION

Missiles are military rocket weapons having an automatic control system to locate its targets or adjust its direction [1]. The first type of missiles used in an operation were German guided missiles in World War II. The most famous were the V-1 and V-2, and both used a simple autopilot system to keep the bullets flying following a predetermined route. In 1962 Rand Paul Barand, from the RAND company, was assigned to develop a decentralized network system capable of controlling its bombing system and missile launching system in a nuclear war.

Lately, several developing and developed countries, especially those with very high levels of aerospace and military technology, have developed several destructive weapons ranging from those user friendly (easy to use), and helpful for people, up to those dangerous and deadly. Examples of user friendly weapons are the creation of military robots functioning to defuse bombs, unmanned monitoring aircrafts, and the likes. Whereas, those of dangerous weapons are the ones generally developed for national defense, such as guided missiles equipped with nuclear or biological weapons.

Indonesia itself, which is a country of archipelago, covers air area of its largest territory, followed by sea area and land area. Logically, the existence of missile defense equipment (the main weapon system) or precisely the type of long-range missile is acceptable to support the defense and security of the Republic of Indonesia, but its consequences to be operated in the territory of Indonesia itself, in case of an occufanct of an error in targeting the target, will fall on of harm to its own national territory [2].

Therefore, trajectory estimation for guided missiles is the basic requirement for guided missiles to be aimed at the precise targets. The trajectory is used as a guide to direct that the missile reach the target by following the given trajectory. Several studies have been carried out regarding the estimation of positions such as the one implemented to AUV using Ensemble and Extended Kalman Filter [3, 4, 5] and Square Root Ensemble Kalman Filter (SR-EnKF) method [6] [7]and Kalman Filter with fuzzy [8], and that implemented on temperature steam drum estimation [9]. Position estimation has also been applied to ASV using the Extended Kalman Filter method [10] To maintain the accuracy of the trajectory continuously, in this study, the missile trajectory estimation was made by using Unscented Kalman Filter (UKF) Algorithm. The contribution of this paper is numeric computation of the of missile position estimation and the missile motion influenced by target

## 2. RESEARCH METHODS

### 2.1 Mathematical Model of Missile Trajectory

Movement equation of missile are modeled as bellow [11, 12]

$$\dot{\gamma} = \frac{1}{mV}\left(L + T\sin\ \alpha\right) - \frac{g}{V}\cos\ \gamma \tag{1}$$

$$\dot{V} = \frac{1}{m}\left(T\cos\ \alpha - D\right) - g\sin\ \gamma \tag{2}$$

$$\dot{x} = V\cos\ \gamma \tag{3}$$

$$\dot{h} = V\sin\ \gamma \tag{4}$$

Where variables state such as flight path angle $\gamma$, speed $V$, horizontal position $x$, and altitude $h$ from missile. A push force $T$ and angle of attack $\alpha$ are two controled variables (see Figure 1). The aerodynamics force $D$ and $L$ are functions the altitude $h$, velocity $V$, and angle of attack $\alpha$. With $L$ is the lift aerodynamic force and $D$ *is* the drag lift aerodynamic force with rescpect to a body-axis frame.

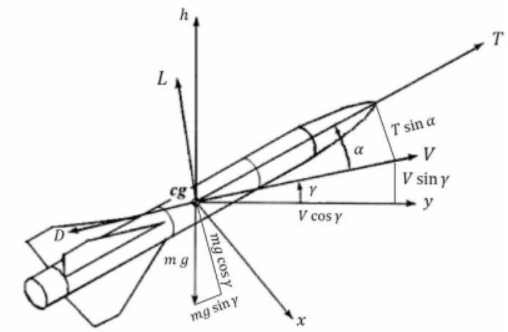

**Figure 1 Axes and Angle Missile Model** [11, 13]

**Aerodynamics Force** [12]

$$D(h,V,\alpha)=\frac{1}{2}C_d\rho V^2 S_{ref} \tag{5}$$

$$C_d = A_1\alpha^2 + A_2\alpha + A_3 \tag{6}$$

$$L(h,V,\alpha)=\frac{1}{2}C_l\rho V^2 S_{ref} \tag{7}$$

$$C_l = B_1\alpha + B_2 \tag{8}$$

with $\rho$ is air density given by

$$\rho = C_1 h^2 + C_2 h + C_3 \tag{9}$$

and $S_{ref}$ is reference area of the missile , $m$ is mass and $g$ is gravitational constant. The value of $A_1$, $A_2$, $A_3$, $B_1$, $B_2$, $C_1$, $C_2$ and $C_3$ are constant given in Table 1.

**Table 1. Physical Modelling Parameter [13]**

| Quantity | Value | unit |
|---|---|---|
| $m$ | 1005 | $kg$ |
| $g$ | 9.81 | $m/s^2$ |
| $S_{ref}$ | 0.3376 | $m^2$ |
| $A_1$ | -1.9431 | |
| $A_2$ | -0.1499 | |
| $A_3$ | 0.2359 | |
| $B_1$ | 21.9 | |
| $B_2$ | 0 | |
| $T$ | 6000 | |
| $C_1$ | $3,312.10^{-9}$ | $\frac{kg^2}{m}$ |
| $C_2$ | $1,142.10^{-4}$ | $\frac{kg^2}{m}$ |
| $C_3$ | 1.224 | $\frac{kg^2}{m}$ |

Because the system requires discretitation, so the missile model in equation (1) – (4) must be discreted using the finite difference method.

Equation (1) – (4) , If $\gamma_k$ flight path angle $k\Delta t$ and identically for speed, horizontal position and altitude $h$ are

$$\gamma = \gamma_k \,;\; V = V_k \;\;;\; x = x_k \;\;;\;\; h = h_k \tag{10}$$

The change of state variables respect to the time are approximated by forward scheme of finite difference. Thus we will get

$$\dot{\gamma} = \frac{d\gamma}{dt} \approx \frac{\gamma_{k+1} - \gamma_k}{\Delta t} \tag{11}$$

$$\dot{V} = \frac{dV}{dt} \approx \frac{V_{k+1} - V_k}{\Delta t} \tag{12}$$

$$\dot{x} = \frac{dx}{dt} \approx \frac{x_{k+1} - x_k}{\Delta t} \tag{13}$$

$$\dot{h} = \frac{dh}{dt} \approx \frac{h_{k+1} - h_k}{\Delta t} \tag{14}$$

from equation (11) – (14) will be gotten the modified missile model in (15) below

$$\begin{bmatrix} \gamma_{k+1} \\ V_{k+1} \\ x_{k+1} \\ h_{k+1} \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{mV_k}(L_k + T_k \sin\ \alpha_k) - \frac{g}{V_k}\cos\ \gamma_k\right)\Delta t + \gamma_k \\ \left(\frac{1}{m}(T_k \cos\ \alpha_k - D_k) - g \sin\ \gamma_k\right)\Delta t + V_k \\ (V_k \cos\ \gamma_k)\Delta t + x_k \\ (V_k \sin\ \gamma_k)\Delta t + h_k \end{bmatrix} \tag{15}$$

## 2.2 Unscented Kalman Filter Algorithm

Algorithm of Unscented Kalman Filter is written as follows [14, 15, 16]:

- *Initiation at $k = 0$:*

  $\hat{x}_0 = E[x_0]$
  $P_{x_0} = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]$
  $\hat{x}_0^a = E[x^a] = E[\hat{x}_0{}^T \;\; 0 \;\; 0]^T$

$$P_0^a = E[(x_0^a - \hat{x}_0)(x_0^a - \hat{x}_0)^T] = \begin{bmatrix} P_x & 0 & 0 \\ 0 & P_v & 0 \\ 0 & 0 & P_n \end{bmatrix} \tag{16}$$

*For $k = 1,2,3,\ldots,\infty$:*

1) Count sigma point

   $X_{k-1}^a = [\hat{x}_{k-1}^a \quad \hat{x}_{k-1}^a + \gamma\sqrt{P_{k-1}} \quad \hat{x}_{k-1}^a - \gamma\sqrt{P_{k-1}}]$
   Dimana:
   $\gamma = \sqrt{L + \lambda}$

$$\lambda = \alpha^2(L + \kappa) - L \tag{17}$$

2) Time-update (prediction stage)

   $X_{k|k-1}^x = f(X_{k-1}^x, X_{k-1}^v)$

$$\hat{x}_k^- = \sum_{i=0}^{2L} W_i^{(m)} X_{i,k|k-1}^x$$

$$P_{x_k}^- = \sum_{i=0}^{2L} W_i^{(c)}(X_{i,k|k-1}^x - \hat{x}_k^-)(X_{i,k|k-1}^x - \hat{x}_k^-)^T$$

$$Z_{k|k-1} = H(X_{k|k-1}^x, X_{k-1}^n)$$

$$\hat{z}_k^- = \textstyle\sum_{i=0}^{2L} W_i^{(m)} Z_{i,k|k-1} \tag{18}$$

3) Measurement update (correction stage):

$$P_{\tilde{z}_k,\tilde{z}_k} = \sum_{i=0}^{2L} W_i^{(c)}(Z_{i,k|k-1} - \hat{z}_k^-)(Z_{i,k|k-1} - \hat{z}_k^-)^T$$

$$
\begin{aligned}
&P_{x_k,z_k} = \sum_{i=0}^{2L} W_i^{(c)} \left(X_{i,k|k-1}^{x} - \hat{x}_k^{-}\right)\left(Z_{i,k|k-1} - \hat{z}_k^{-}\right)^T \\
&K_k = P_{x_k,z_k} P_{\bar{z}_k,\bar{z}_k}^{-1} \\
&\hat{x}_k = \hat{x}_k^{-} + K_k\left(z_k - \hat{z}_k^{-}\right) \\
&P_{x_k} = P_{x_k}^{-} - K_k P_{\bar{z}_k} \mathrm{K}_k^T
\end{aligned}
\qquad (19)
$$

## 3. RESULTS AND DISCUSSION

Numerical computation in this paper makes the missile model a platform in missile trajectory estimation because the missile model is a nonlinear model. So, the use of UKF is one way to obtain a high accuracy. In this paper, two simulations are compared, covering a simulation on the missile trajectory for going upwards and then plunging downwards, while the second simulation is a missile shooting a target at an altitude of about 1000 meters. The first simulation is represented in Figure 2 – Figure 5, and the second simulation is represented by Figure 6 – Figure 9 of error value for two simulations is in Table 2.

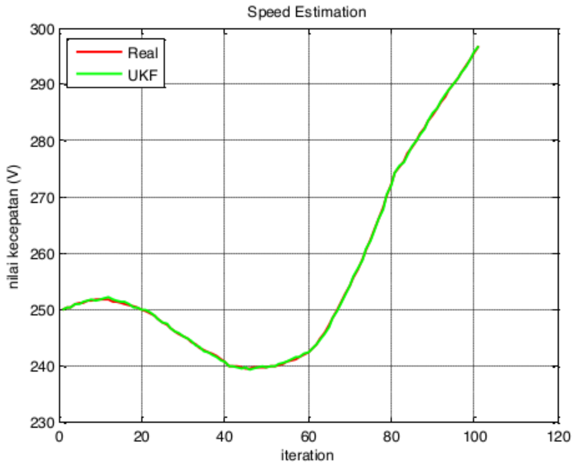

**Figure 2**. **Speed Estimation of Missile in the first trajectory using UKF**

Figure 2 indicates that the speed slightly increased from 250 m/s to 254 m/s in the 10th iteration and then decreased to 240 m/s in the 40th iteration, then increased again to 270 m/s in the 80th iteration, and increased rapidly to 295 m/s in the 100th iteration. In this case the missile speed decreases when it goes up, but when it plunges down, its speed increases. Velocity is closely related to horizontal position (x) and altitude (h), increasing and decreasing speed is affected by horizontal position (x) and altitude (h).

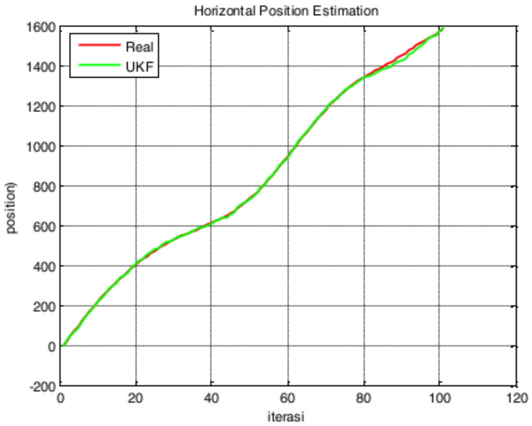

**Figure 3**. **Horizontal Position Estimation of Missile in the first trajectory using UKF**

Figure 3 shows that the horizontal position of the missile advances to a distance of 1600 meters and undergoes a slight bend during the 50th and 70th iterations and returns running straight up to the 100th iteration with a distance of 1600 meters. In this case the horizontal position of the missile is always forward and will not return (turn back). The horizontal position is closely related to the height at which when the horizontal position is advancing (not turning) them the height will follow the horizontal position to get the maximum height and plunge down.

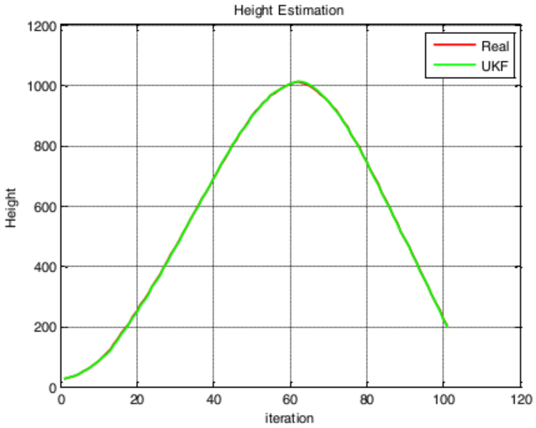

**Figure 4. Height Estimation of Missile in the first trajectory using UKF**

Figure 4 shows that the missile always rise up to the turning point of 1000 meters in the 62nd iteration and finally plunge down. In this case Figure 4 and Figure 3 are closely related because when the missile goes up the speed is adjusted by slightly increasing and decreasing the speed, but when it is in the 62nd iteration which is the turning point, the speed increases rapidly when it plunges down. In this case the height of the missile represents an increase when flying, and it represents a decrease when it plungs down to the ground.

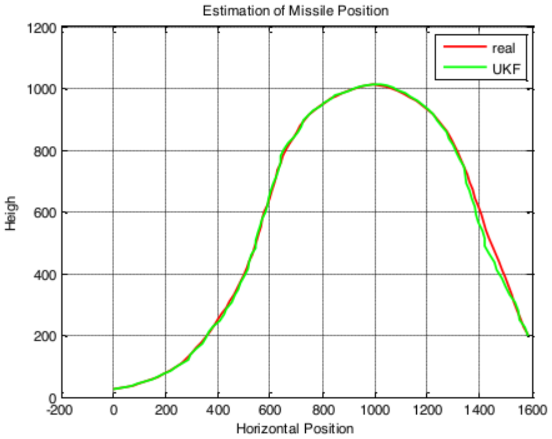

**Figure 5**. **Trajectory Estimation of Missile in the first trajectory using UKF**

In Figure 5 it can be seen that the guided missile always rises to a height of 1000 meters with a distance of 1000 meters from the start of firing, and finally plunges down from an altitude of 900 meters to the ground with a distance of 1600 meters from the start of firing. In this case, the relationship between the horizontal position and the height of the missile is about how far the missile travels at what height will it strike and when it hits the ground. Next, it is the simulation with the second trajectory shown in Figure 6 – figure 9.

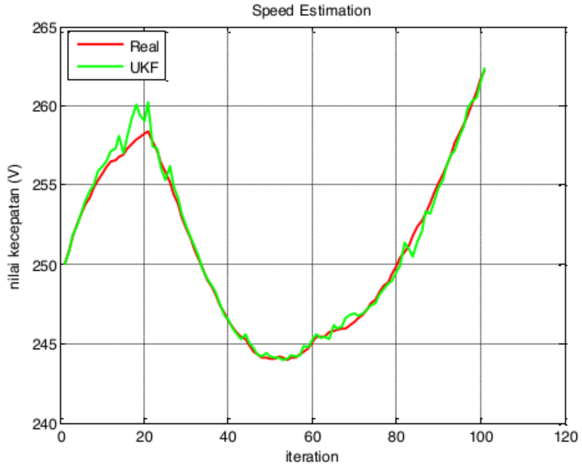

**Figure 6**. Speed Estimation of Missile in the Second trajectory using UKF

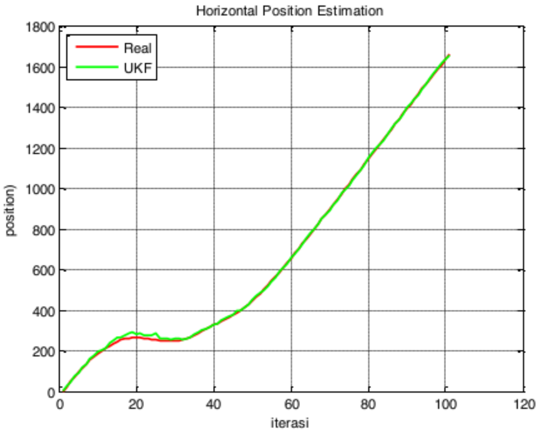

**Figure 7**. Horizontal Position Estimation of Missile in the Second trajectory using UKF

Similar to Figure 2 and 3, Figure 6 and 7, they show that the UKF method has a near perfect accuracy of about 97%. Here can it be observed that between the actual and estimated results of trajectories coincide with each other.

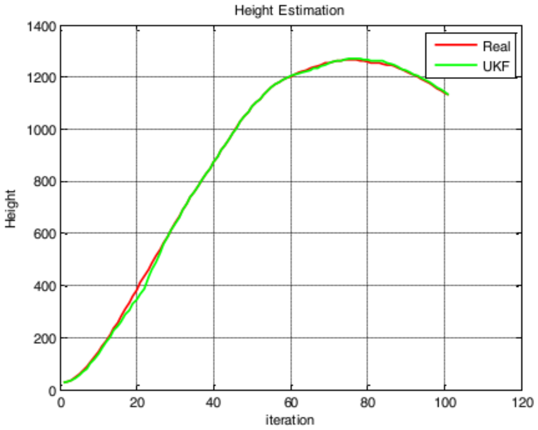

**Figure 8**. Height Estimation of Missile in the Second trajectory using

Figures 8 and 9 show that the red line is the real horizontal position, and the height that is determined to shoot a target flying at a certain height and distance so that the missile hits the target aimed at, in which for instance the target is at an altitude of 1100 meters and a distance of 1600 meters.

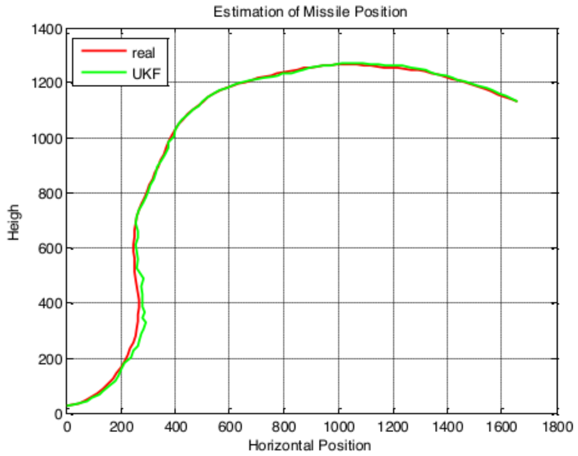

**Figure 9**. Trajectory Estimation of Missile in the Second trajectory using UKF

In table 2 it can be seen that both the first and second simulations have almost the same accuracy when using the UKF method, but the first simulation, in which the missile is fired up and down, has a smaller error than the second simulation, which is a simulation of striking a target from a certain distance and altitude.

**Table 2**. RMSE value from Result Computational Simulation

| | UKF | |
|---|---|---|
| | RMSE First Trajectory | RMSE Second Trajectory |
| Angle Position | 0.26501 | 0.39078 |
| Speed | 0.12113 | 0.2774 |
| Horizontal Position | 0.86408 | 0.9278 |
| Height | 0.91935 | 0.9649 |

## 4. CONCLUSION

Based on the simulation analysis, some conclusions are present: Unscented Kalman Filter (UKF) method can be applied to estimate the missile trajectory and that both the first and second simulations have almost the same accuracy when using the UKF method, but the first simulation, in which the missile is fired up and down, has a smaller error than the second simulation, which is a simulation of striking a target from a certain distance and altitude.

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